The Fan Calculus

Ontology for the Future of AI

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Contents

40 years wrestling with the foundations of AI . . .

1) **Mechanism** · The nature of **computing**

2) **Semantics** · The nature of **representation**

✔ 3) **Ontology** · The nature of **objects** (and other stuff)

For the record: Classical Ontology

1. A world of **objects** ⇐ referred to with **names/terms**

   a) exemplifying **properties** ⇐ described with **predicates**

   b) standing in **relations** ⇐ described with **relation terms**

   c) grouped together in **sets** ⇐ referred to with **plurals**

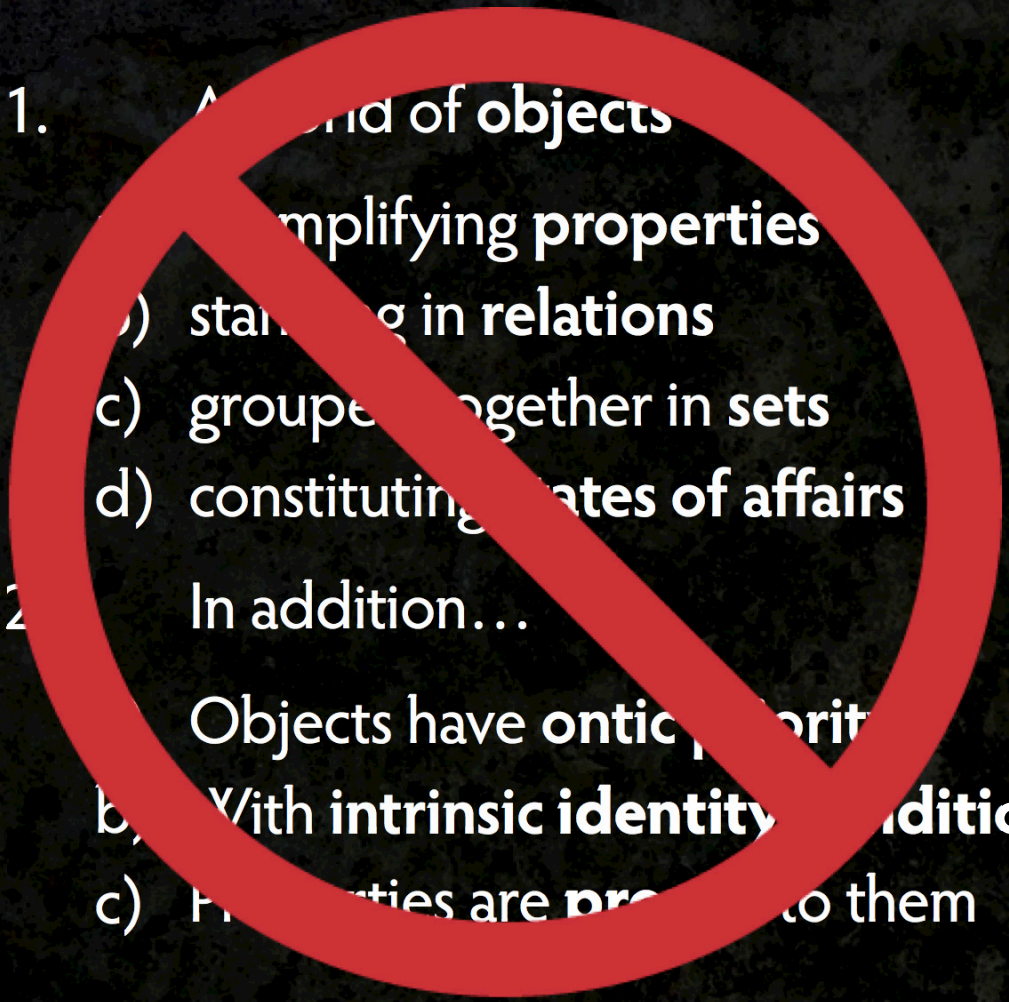
   d) constituting **states of affairs** ⇐ designated by **sentences**

ontology **epistemology**

For the record: Classical Ontology

1. A world of **objects**
   a) exemplifying **properties**
   b) standing in **relations**
   c) grouped together in **sets**
   d) constituting **states of affairs**
2. In addition…
   a) Objects have **ontic priority**
   b) With **intrinsic identity conditions**
   c) Properties are **proper** to them

For the record: Classical Ontology



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   a) amplifying **properties**
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2. In addition…
   a) Objects have **ontic priority**
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   c) Properties are **prior** to them

Problems

1. Issues of **non-conceptual content**
2. Issues of the **background**
3. Issues of **ambiguity** and **equivocation**
4. ✔ Issues even with respect to **objects**

   — including what I want to talk about today: an issue that has been wrestled with for thousands of years

   — the issue of **the one and the many**

Complex objects

1. **Documents**

   — works, versions, editions, translations, copies, etc.

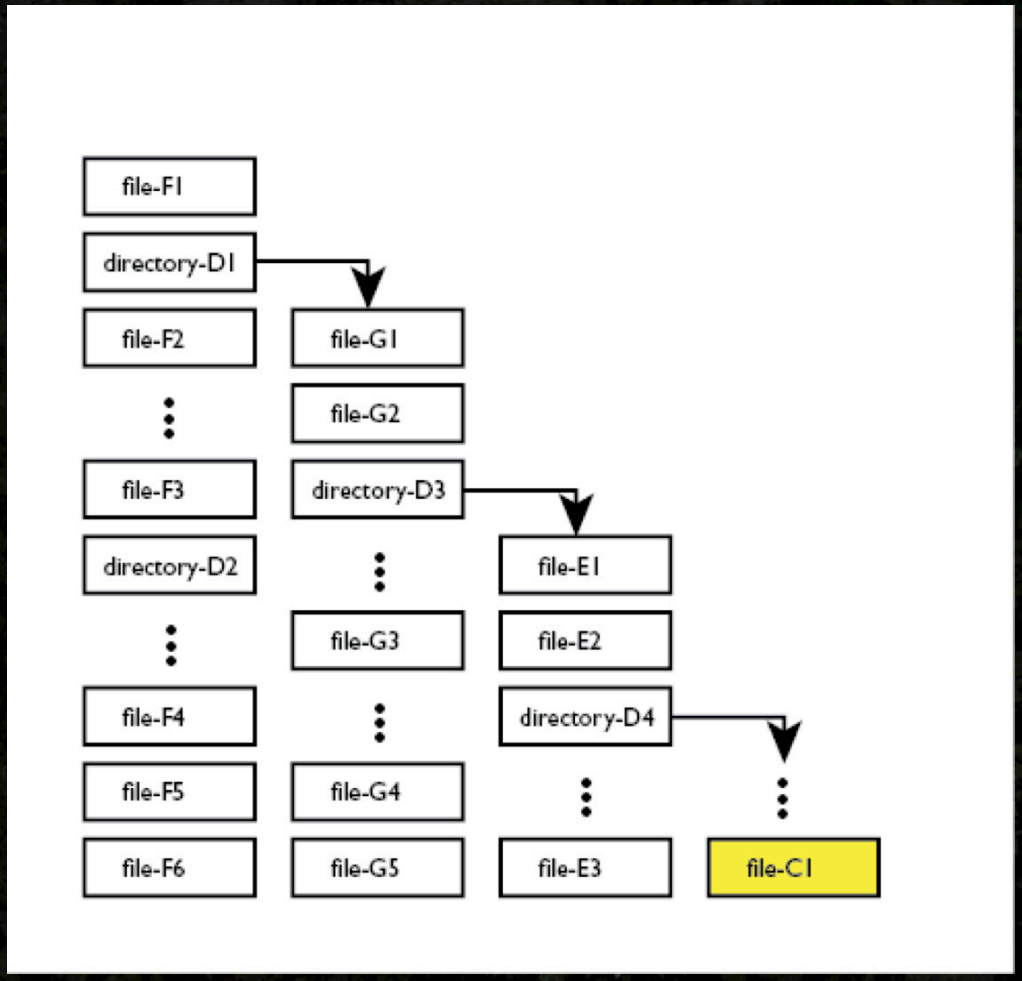
   — cf. FRBR (150 page report)

2. **Files**

   — in memory, in cache, on disk, the "same file" on a different disk

   — "Yuck! This file is corrupted! It was OK a few moments ago. Fortunately, I backed it up last night. But I've changed it a lot since then. But maybe I'll remember enough, so that if I retrieve it, I'll be able to fix it up in a couple of hours."

   — 5 references; all different!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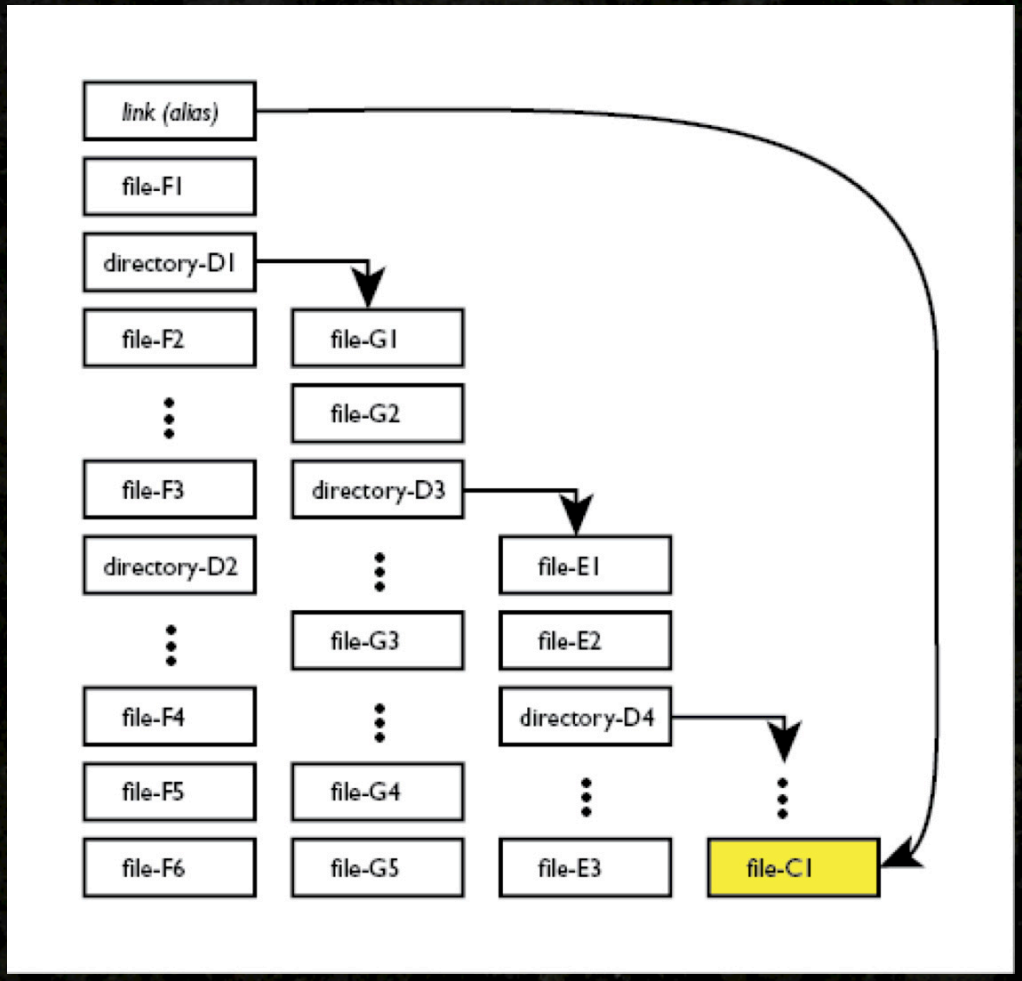
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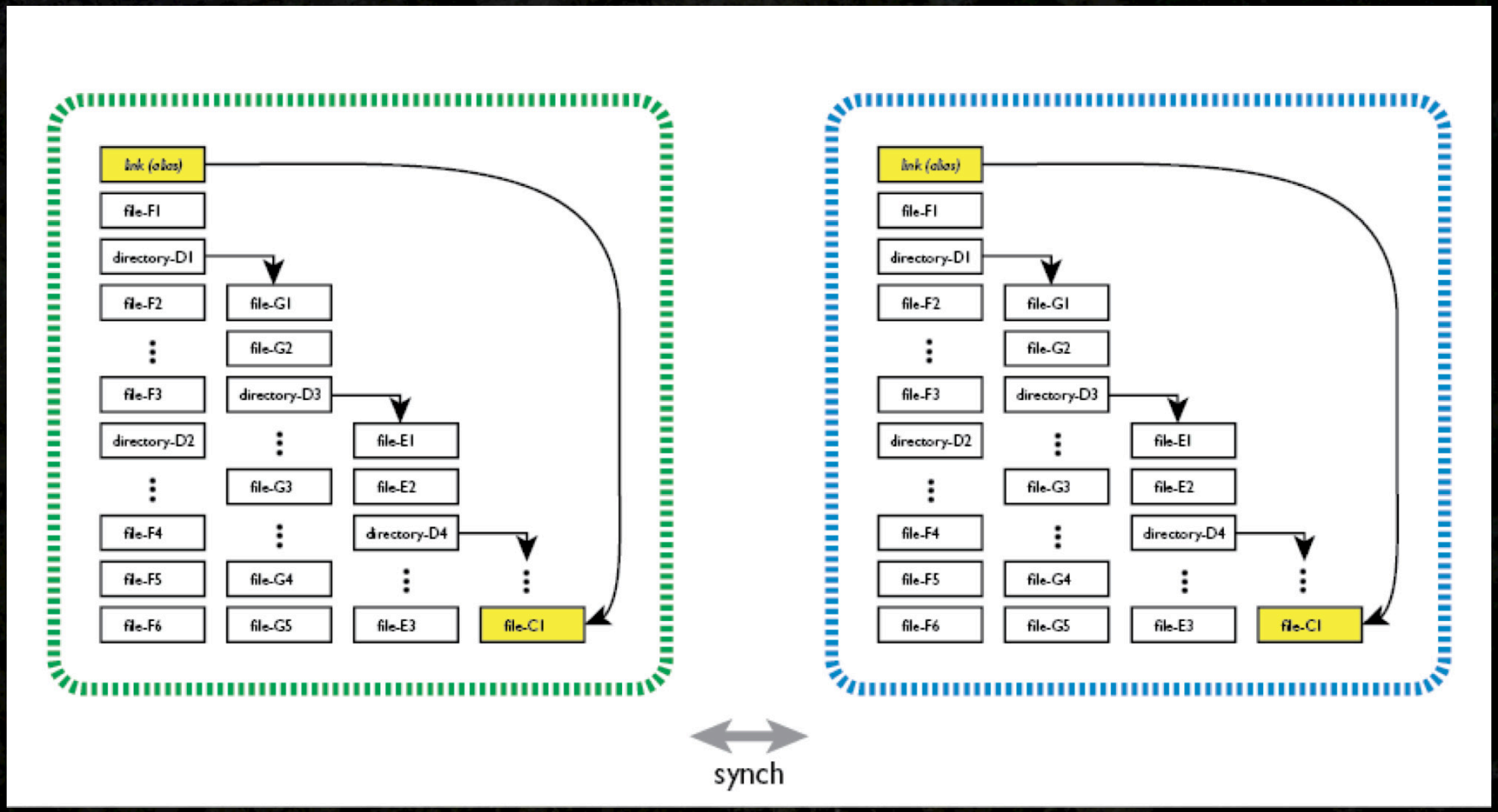
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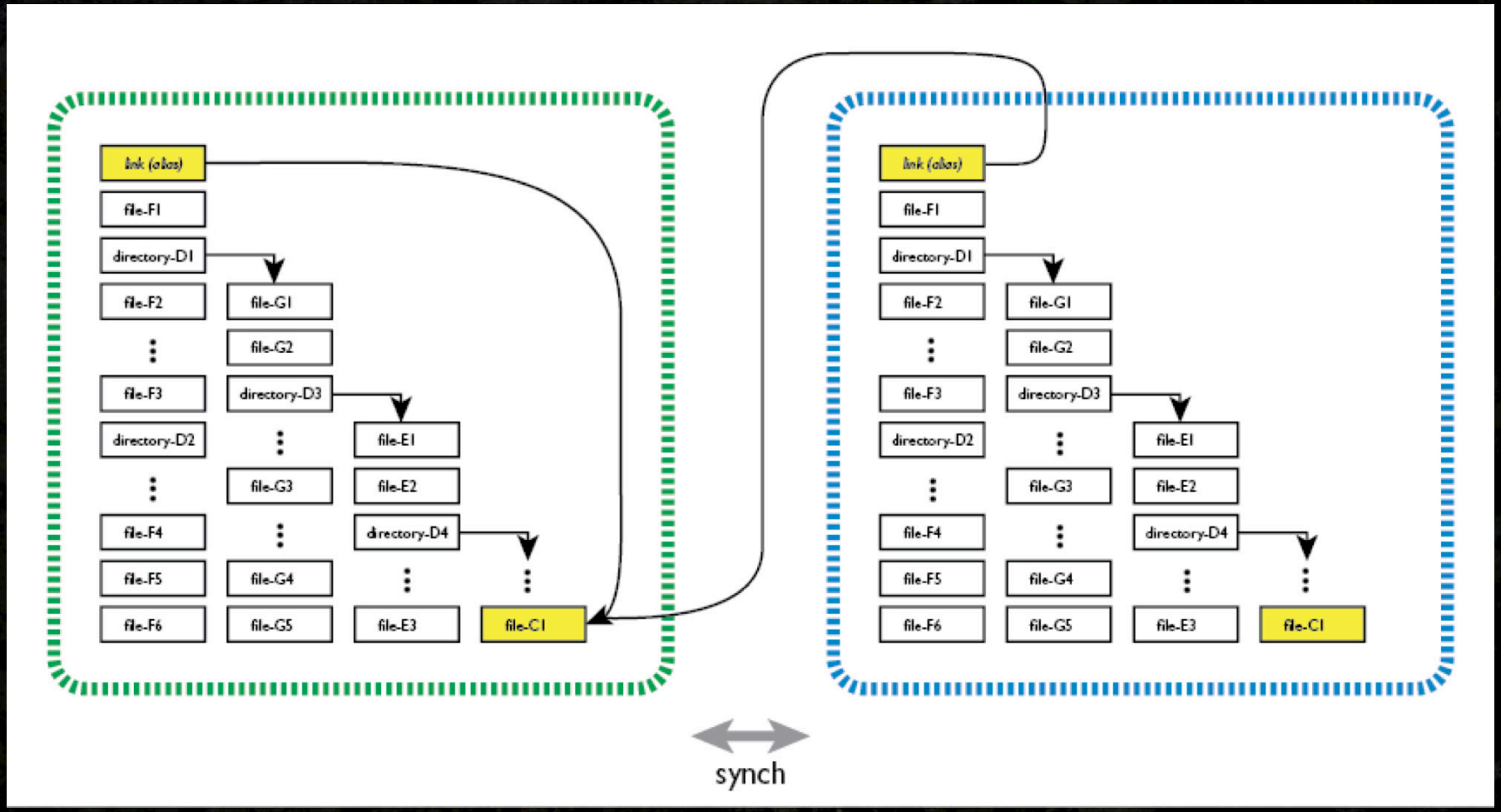
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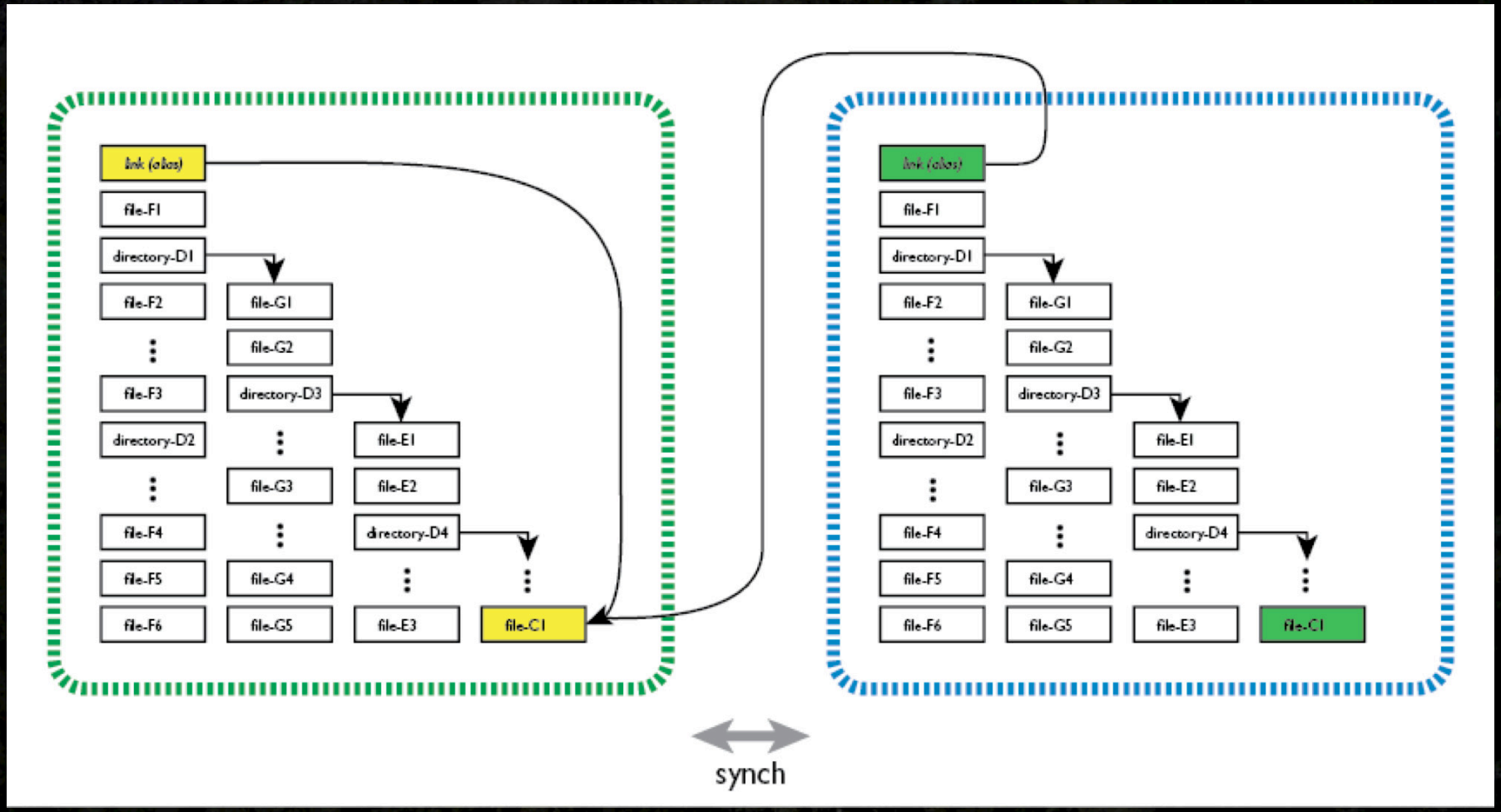
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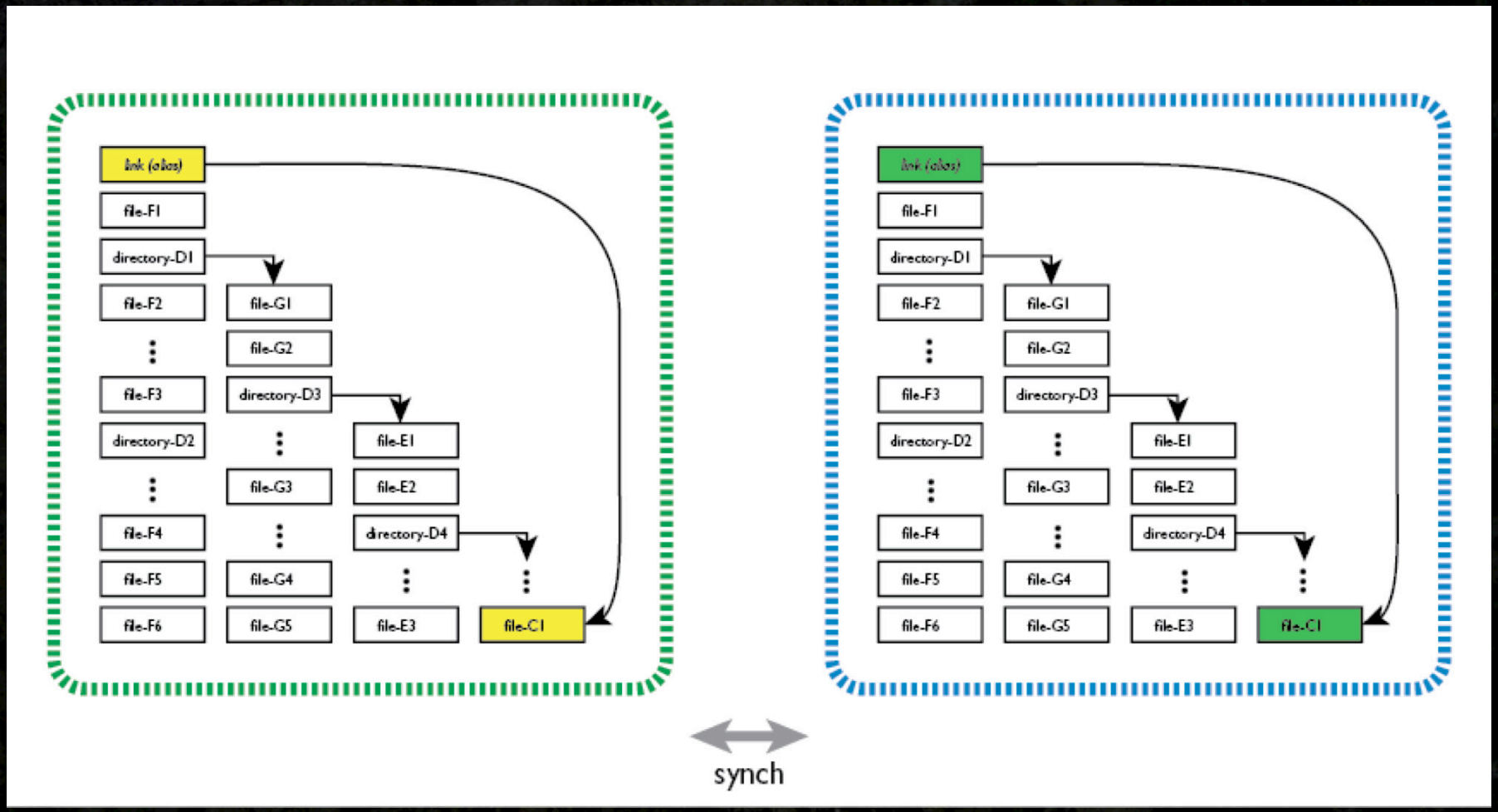
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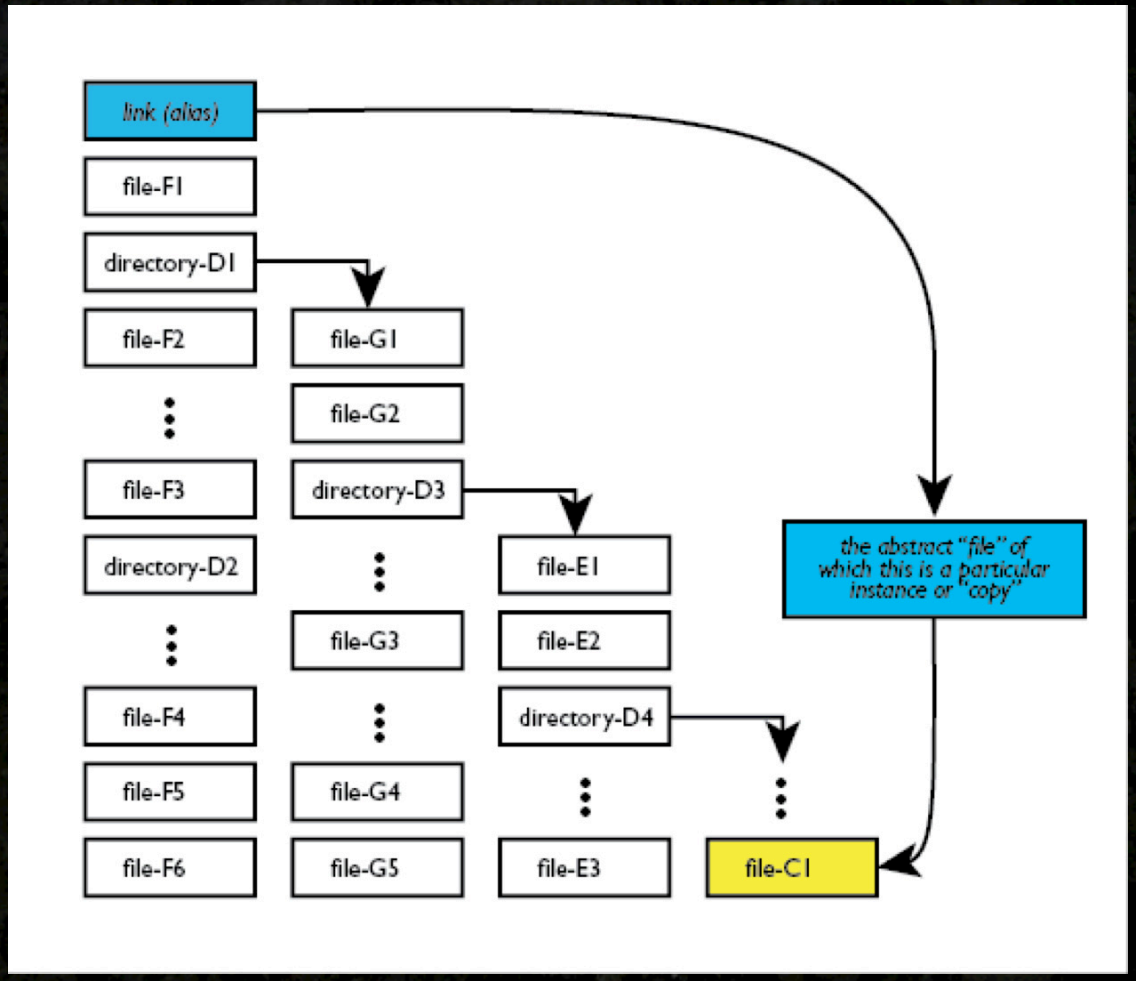
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the abstract "file" of which this is a particular instance or "copy"

Not just "techy" examples

3. **Names**

— "One name — one person"

4. **Chairs** (of a committee)

✔ The chair is usually an anthropologist

✔ The chair is retiring in June

✘ The chair is usually an anthropologist and is retiring in June

— gets very **zeugmatic**

But not always zeugmatic

1. **Example**

   — You are reading a book, and a friend says "Did you know that the author of that book lives off the grid?"

   — "This one?", you ask, holding up a torn paperback.

   — "Yeh, the torn one" your friend says.

2. Note that there is **nothing problematic** about that conversation

   — It doesn't seem zeugmatic at all.

   — Why not?

A Bad Idea

1. **Codify** all of the various possibilities
   - — Philosophy: types, tokens, instances, utterances
   - — Libraries (FRBR): works, editions, manifestations, items
2. But this is **hopeless**
   - — One can't know all the possibilities in advance
   - — Any attempt to do so is (unusably) baroque & brittle
   - — One attempt (at this ontic profusion) won't be the same as another

A Better Idea

1. Metaphysical world view explored in "On the Origin of Objects"
2. Subjects **register** the world in terms of normative considerations pertinent to the material projects in which they are engaged.
   a) A subject ("registrar") **registers an apple**, or **registers confusion**
   b) We register what is **significant** to our projects
3. Leads to a different view of objects: as:
   a) A **material object** is a **patch of reality that matters**

Identity

1. Leads to a different conception of **object identity**.
2. Identity is
   a) **Contextual**
   b) **Contested**
   c) **Dynamic**
   d) **Fluid**

   } a propos to the project at hand (ready to / present at)
3. That is, identity is a
   a) **Dynamic, perspectival** matter of how stuff is referred to
   b) Not a **static matter of intrinsic fact**

Contents

Calculus

1. It's not enough merely to say that identity is perspectival, contextual, fluid, etc.
2. It would be good to know **how it actually works in this way**
3. Leads to a proposal to construct ✔

   A calculus of the one and the many
4. Aim is to

   a) Embody an understanding of how, in fact, we register singly and plurally; and

   b) Serve as a basis in terms of which to build and analyse intelligent objectifying behaviour (behaviour of registering

$\Gamma \vdash_P \forall x A(x) \Leftrightarrow . \forall \Gamma^* \Gamma^* \vdash_P A(t)$ for all $t \in P(\Gamma^*)$.

The Gang of Six

1. Radix (positional) arithmetic, with zero
2. Algebra
3. "The (differential) calculus"
4. Set theory
5. Quantificational logic
6. The λ-calculus

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  326
  947
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1,273
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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

$$\int_0^N \int_0^N e^{-xy} \sin x \, dx \, dy = \int_0^N \frac{1}{1+y^2} - \frac{\cos N + y \sin N}{e^{Ny}(1+y^2)} \, dy.$$

$$\bigcup_{\gamma \in \Gamma} A_\gamma = \{x : \exists \gamma \in \Gamma (x \in A_\gamma)\},$$

$$\bigcap_{\gamma \in \Gamma} A_\gamma = \{x : \forall \gamma \in \Gamma (x \in A_\gamma)\}.$$

- $((\lambda x.x\ x)(\lambda y.y)) \rightarrow_\beta ((\lambda y.y)(\lambda y.y)) \rightarrow_\beta (\lambda y.y)$
- $((\lambda x.(\lambda y.x\ y))y) \rightarrow_\beta (\lambda y_0.y\ y_0)$

Discussion

1. Every one of the six **builds in** the classical model of ontology
   — determinate, intrinsic, absolute identity
2. Every one also is classical **with respect to its own ingredients**
3. These points are true of all current formal systems of which I know
   — including RDF, OWL, XML, common logic, data bases…
4. So if we are going to embrace the "**principle of perspectival identity**", we need something else

Division of Labour

1. Newton/Leibniz
   a) How much (achievement) in the **formulation of the calculus**?
   b) How much in the formulation of the laws of motion **in the calculus**?
2. What content
   a) Is in the architecture or language itself (primitives and categories)?
   b) Can be expressed in that architecture of language?
   c) Cannot be said at all?
3. The gang of six
   a) Build their accounts of the many-one into the architecture
   b) Functions/values, sets/members, quantification, etc.

Principles for the fan calculus

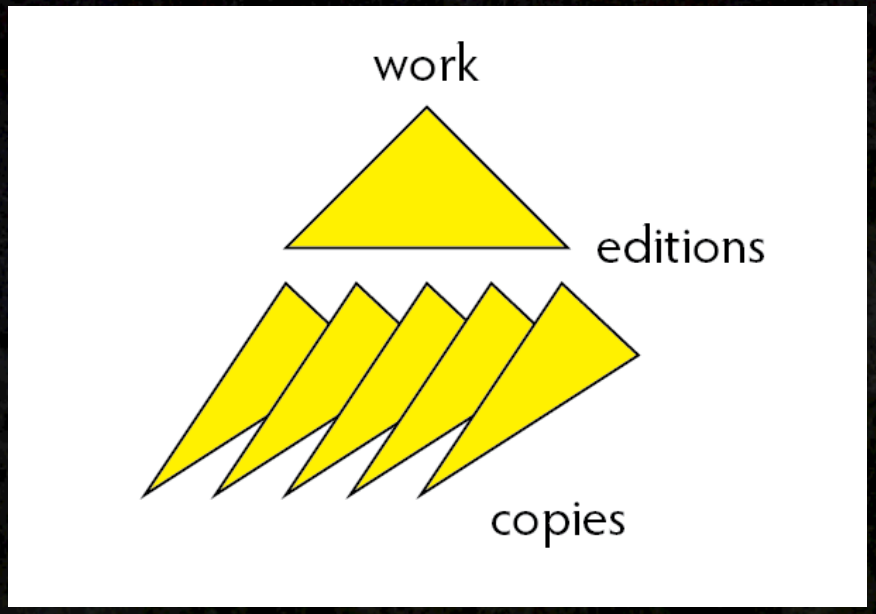
1. Identity as perspectival fact of how stuff is registered (not intrinsic)
2. No specifics about one–many to be syncategorematic
3. … many others (about semantics, reflection, etc.)

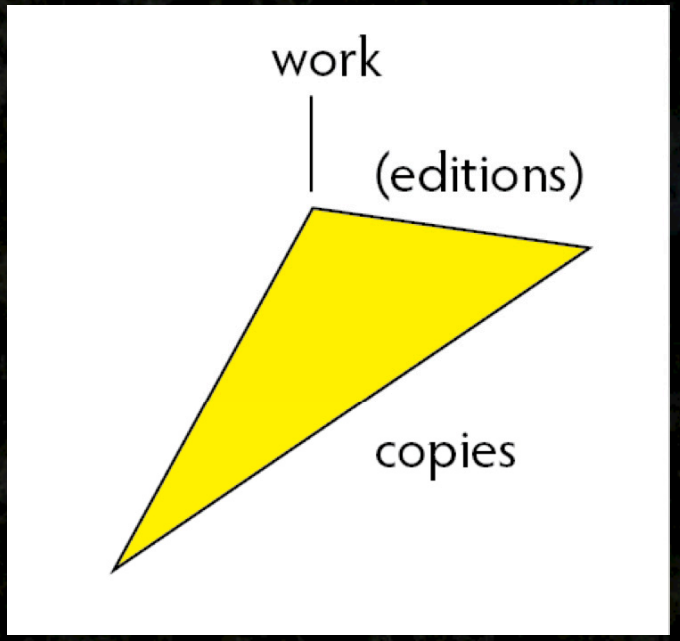
Contents

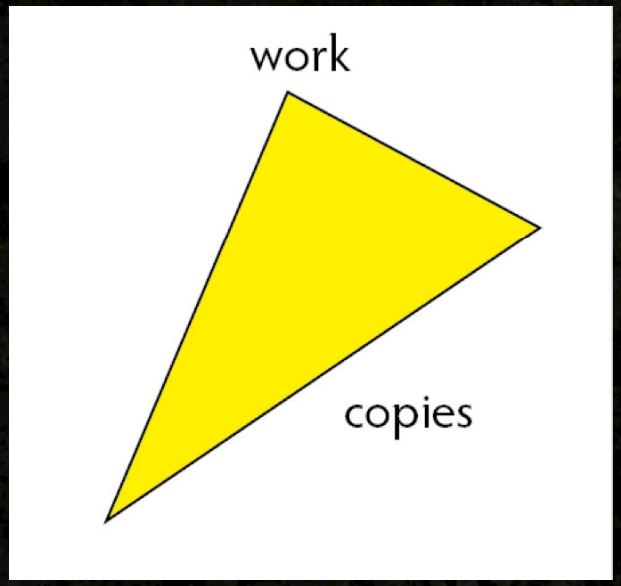
Basic insight

1. Distinctions (Δs) made *when germane*
   a) "Break open" what would otherwise be one into several/many
   b) "Fan-out"
2. If the Δ is not germane, "collapse" the Δ (fan), so that there is no fact of the matter, wrt the Δ, as to what one is talking about
   a) I.e., treat is as a unity (singly)

Example: documents

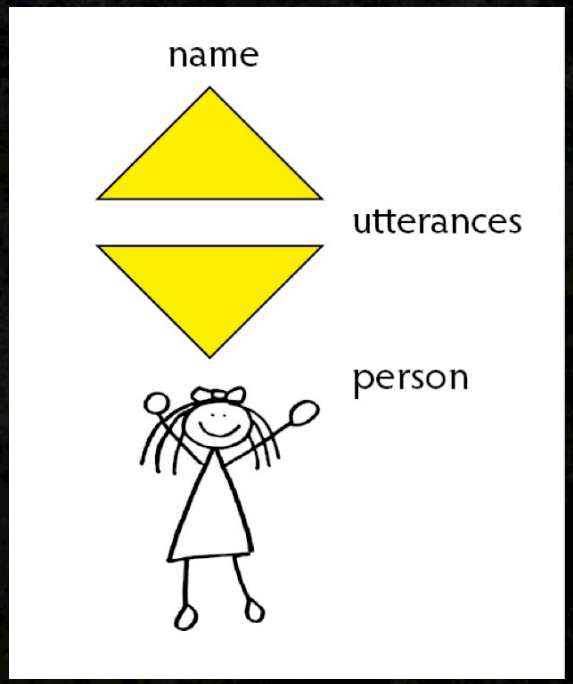






work = edition = copy

Example: names



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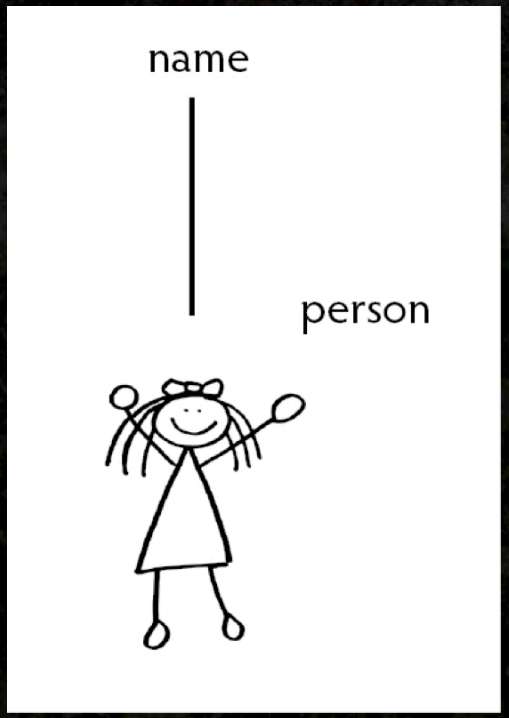

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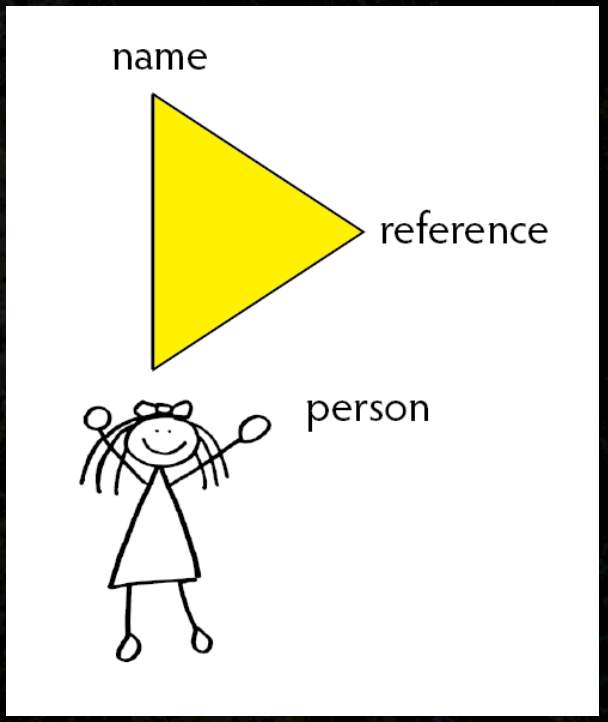

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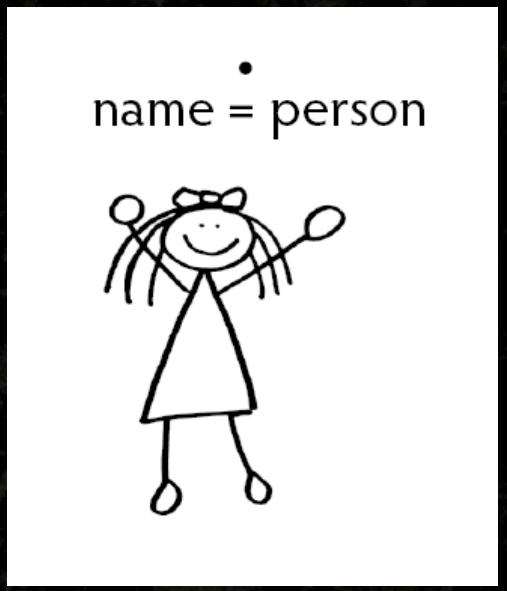
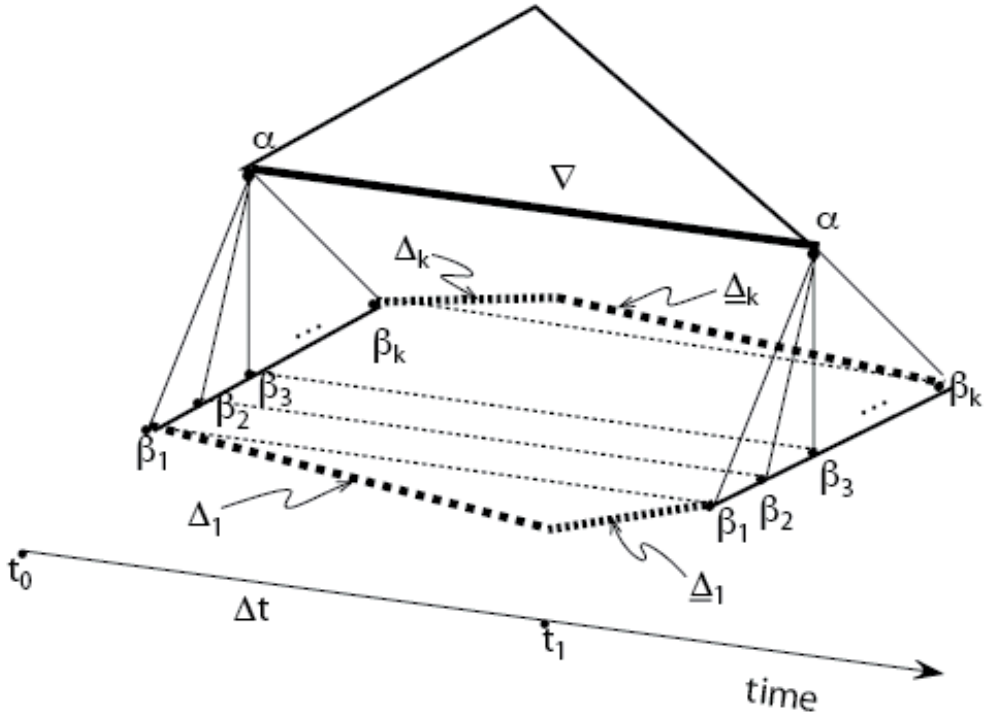


(d)

Abstraction & concretization

1. The **distinguishing principle** for each fan describes what is
   a) **Abstracted away from**, at the "top" (the entity as singular)
   b) **Concretized** — brought into view, at the "bottom" (as plural)
2. Leads to "**abstraction**" and "**concretization**" operations
3. Example: words and their spellings
   a) "Aluminum" on North American versions of a web site
   b) "Aluminium" on British

Example: file synchronization



$$\underline{\Delta}_k = \downarrow_k \mathbf{U}(\uparrow \Delta_i) - \Delta_k$$

In sum

1. A **constructive, recursive, descriptive, and reflective** calculus
2. **Identity** (including of the calculus' elements themselves) taken to be a perspectival matter of contingency and perspective, rather than an intrinsic property of objects
3. Distinctions (Δs, fans) only opened up only **when and if relevant**
   a. type/token
   b. whole/part
   c. set/member
   d. original/copy
   e. work/edition/version/item…
   f. file/copy…
   g. name/named (sign/signified)
   h. … etc.
4. That said, **I really have no idea how this calculus is going to work**